

Limits on the Mass of a Composite Higgs Boson^a

R. Sekhar Chivukula

*Department of Physics, Boston University, 590 Commonwealth Ave.,
Boston, MA 02215 USA*



We discuss the bound on the mass of the Higgs boson arising from precision electroweak measurements in the context of the triviality of the scalar Higgs model. We show that, including possible effects from the underlying nontrivial dynamics, a Higgs boson mass of up to 500 GeV is consistent with current data.

Current results from the LEP Electroweak Working Group^{2,3} favor a Higgs boson mass that is relatively light. The “best-fit” value for the Higgs mass is somewhat less than experimental lower bound of 107.7 GeV reported at this conference.⁴ The 68% and 95% CL upper bounds from precision measurements, in the context of the standard model, are 127 and 188 GeV respectively. It is possible that, as these data suggest, the Higgs boson lies around the corner and will be discovered at relatively low masses. On the other hand, it is important to consider alternatives and to understand the wider class of models consistent with precision electroweak tests. In this talk, I will show that even minor modifications to the standard electroweak theory allow for a substantially heavier Higgs boson.

1 The Triviality of the Standard Higgs Model

This task is made easier, and is also motivated, by the fact that the standard one-doublet Higgs model *does not strictly exist* as a continuum field theory. This result is most easily illustrated in terms of the Wilson renormalization group.⁶ Any quantum field theory is defined using a regularization procedure which ameliorates the bad short-distance behavior of the theory. Following Wilson, we define the scalar sector of the standard model

$$\mathcal{L}_\Lambda = D^\mu \phi^\dagger D_\mu \phi + m^2(\Lambda) \phi^\dagger \phi + \frac{\lambda(\Lambda)}{4} (\phi^\dagger \phi)^2 \quad (1)$$

^aFor a more complete discussion, see ref. 1 and references therein.

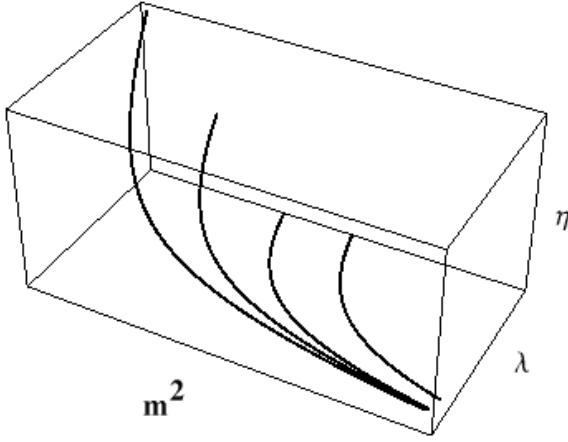


Figure 1: Graphical representation of Wilson RG flow of $(m^2(\Lambda), \lambda(\Lambda), \eta(\Lambda))$. As we scale to low energies, $m^2 \rightarrow \infty$, $\lambda \rightarrow 0$, and $\eta \rightarrow 0$.

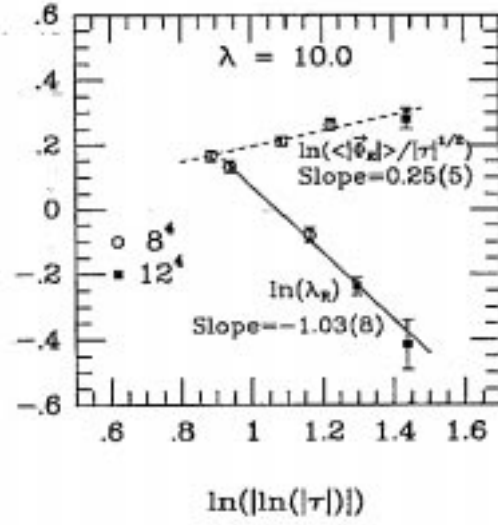


Figure 2: Results of a nonperturbative lattice monte carlo study⁵ of the scalar sector of the standard model with bare coupling $\lambda = 10$. The approximate slope of -1 for the renormalized coupling, λ_R , shows agreement with the naive one-loop perturbative result.

$$+ \frac{\eta(\Lambda)}{36\Lambda^2} (\phi^\dagger \phi)^3 + \dots$$

in terms of a fixed UV-cutoff Λ . Here we have allowed for the possibility of terms of (engineering) dimension greater than four. While there are an infinite number of such terms, one representative term of this sort, $(\phi^\dagger \phi)^3$, has been included explicitly for the purposes of illustration. Note that the coefficient of the higher dimension terms includes the appropriate number of powers of Λ , the intrinsic scale at which the theory is defined.

Wilson observed that, for the purposes of describing experiments at some fixed low-energy scale $E \ll \Lambda$, it is possible to trade a high-energy cutoff Λ for one that is slightly lower, Λ' , so long as $E \ll \Lambda' < \Lambda$. In order to keep low-energy measurements fixed, it will in general be necessary to redefine the values of the coupling constants that appear in the Lagrangian. Formally, this process is referred to as “integrating out” the (off-shell) intermediate states with $\Lambda' < k < \Lambda$. Keeping the low-energy properties fixed we find

$$\begin{aligned} \mathcal{L}_\Lambda &\Rightarrow \mathcal{L}_{\Lambda'} \\ m^2(\Lambda) &\rightarrow m^2(\Lambda') \\ \lambda(\Lambda) &\rightarrow \lambda(\Lambda') \\ \eta(\Lambda) &\rightarrow \eta(\Lambda') . \end{aligned} \tag{2}$$

Wilson’s insight was to see that many properties of the theory can be summarized in terms of the evolution of these (generalized) couplings as we move to lower energies. Truncating the infinite-dimensional coupling constant space to the three couplings shown above, the behavior of the scalar sector of the standard model is illustrated in Figure 1. This figure illustrates a number of important features of scalar field theory. As we flow to the infrared, *i.e.* lower the effective cutoff, we find:

- $\eta \rightarrow 0$ — this is the modern interpretation of renormalizability. If $m_H \ll \Lambda$, the theory is drawn to the two-dimensional (m_H, λ) subspace. Any theory, therefore, in which $m_H \ll \Lambda$ is close to a renormalizable theory with corrections suppressed by powers of Λ .

- $m^2 \rightarrow \infty$ — This is the naturalness/hierarchy problem. To maintain $m_H \simeq \mathcal{O}(v)$ we must adjust^b the value of m_H in the underlying theory to of order

$$\frac{\Delta m^2(\Lambda)}{m^2(\Lambda)} \propto \frac{v^2}{\Lambda^2} . \quad (3)$$

- $\lambda \rightarrow 0$ — The coupling λ has a positive β function and, therefore, as we scale to low energies λ tends to 0. If we try to take the “continuum” limit, $\Lambda \rightarrow +\infty$, the theory becomes free or trivial.⁶

The triviality of the scalar sector of the standard one-doublet Higgs model implies that this theory is only an effective low-energy theory valid below some cut-off scale Λ . Given a value of $m_H^2 = 2\lambda(m_H)v^2$, there is an *upper* bound on Λ . An *estimate* of this bound can be obtained by integrating the one-loop β -function, which yields

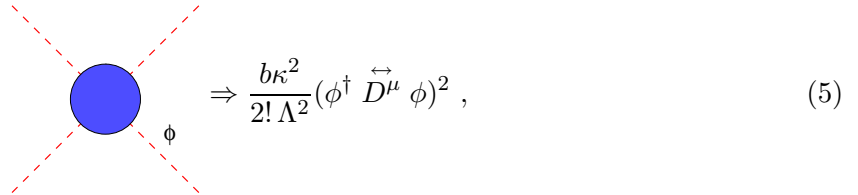
$$\Lambda \lesssim m_H \exp \left(\frac{4\pi^2 v^2}{3m_H^2} \right) . \quad (4)$$

For a light Higgs, the bound above is at uninterestingly high scales and the effects of the underlying dynamics can be too small to be phenomenologically relevant. For a Higgs mass of order a few hundred GeV, however, effects from the underlying physics can become important. I will refer to these theories generically as “composite Higgs” models.

Finally, while the estimate above is based on a perturbative analysis, nonperturbative investigations of $\lambda\phi^4$ theory on the lattice show the same behavior. This is illustrated in Figure 2.

2 T , S , and U in Composite Higgs Models

In an $SU(2)_W \times U(1)_Y$ invariant scalar theory of a single doublet, all interactions of dimension less than or equal to four also respect a larger “custodial” symmetry⁷ which insures the tree-level relation $\rho = M_W^2/M_Z^2 \cos^2 \theta_W \equiv 1$. The leading custodial-symmetry violating operator is of dimension six⁸ and involves four Higgs doublet fields ϕ . In general, the underlying theory does not respect the larger custodial symmetry, and we expect the interaction



$$\Rightarrow \frac{b\kappa^2}{2!\Lambda^2} (\phi^\dagger \overleftrightarrow{D}^\mu \phi)^2 , \quad (5)$$

to appear in the low-energy effective theory. Here b is an unknown coefficient of $\mathcal{O}(1)$, and κ measures size of couplings of the composite Higgs field. In a strongly-interacting theory, κ is expected⁹ to be of $\mathcal{O}(4\pi)$.

Deviations in the low-energy theory from the standard model can be summarized in terms of the “oblique” parameters^{10,11} S , T , and U . The operator in eqn. 5 will give rise to a deviation ($\Delta\rho = \varepsilon_1 = \alpha T$)

$$|\Delta T| = |b|\kappa^2 \frac{v^2}{\alpha(M_Z)\Lambda^2} \gtrsim \frac{|b|\kappa^2 v^2}{\alpha(M_Z^2) m_H^2} \exp \left(-\frac{8\pi^2 v^2}{3m_H^2} \right) , \quad (6)$$

^bNothing we discuss here will address the hierarchy problem directly.

where $v \approx 246$ GeV and we have used eqn. 4 to obtain the final inequality. The consequences of eqns. (4) and (6) are summarized in Figures 3 and 4. The larger m_H , the lower Λ and the larger the expected value of ΔT . Current limits imply $|T| \lesssim 0.5$, and hence $\Lambda \gtrsim 4 \text{ TeV} \cdot \kappa$. (For $\kappa \simeq 4\pi$, $m_H \lesssim 450$ GeV.)

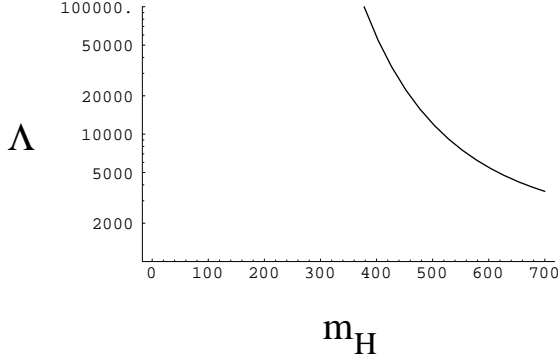


Figure 3: Upper bound on scale Λ as per eqn. (4).

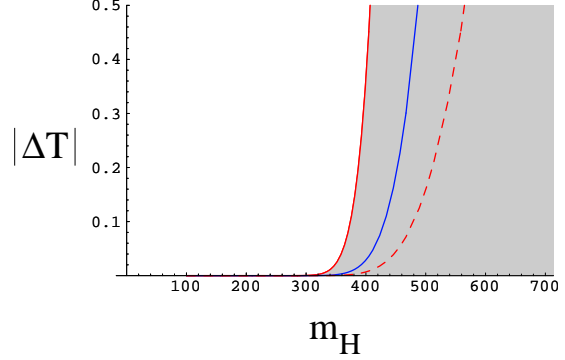
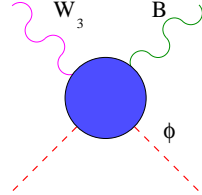


Figure 4: Lower bound on expected size of $|\Delta T|$ as per eqn. (6), for $|b|\kappa^2 = 16\pi^2$, 4π , and 3.

By contrast, the leading contribution to S arises from



$$\Rightarrow -\frac{a}{2! \Lambda^2} \{[D_\mu, D_\nu] \phi\}^\dagger [D^\mu, D^\nu] \phi. \quad (7)$$

This gives rise to ($\varepsilon_3 = \alpha S / 4 \sin^2 \theta_W$)

$$\Delta S = \frac{4\pi a v^2}{\Lambda^2}. \quad (8)$$

It is important to note that the size of contributions to ΔT and ΔS are very different

$$\frac{\Delta S}{\Delta T} = \frac{a}{b} \left(\frac{4\pi\alpha}{\kappa^2} \right) = \mathcal{O} \left(\frac{10^{-1}}{\kappa^2} \right). \quad (9)$$

Even for $\kappa \simeq 1$, $|\Delta S| \ll |\Delta T|$.

Finally, contributions to U ($\varepsilon_2 = -\frac{\alpha U}{4 \sin^2 \theta_W}$), arise from

$$\frac{c g^2 \kappa^2}{\Lambda^4} (\phi^\dagger W^{\mu\nu} \phi)^2 \quad (10)$$

and, being suppressed by Λ^4 , are typically much smaller than ΔT .

3 Limits on a Composite Higgs Boson

From triviality, we see that the Higgs model can only be an effective theory valid below some high-energy scale Λ . As the Higgs becomes heavier, the scale Λ *decreases*. Hence, the expected size of contributions to T *grow*, and are larger than the expected contribution to S or U . The limits from precision electroweak data in $(m_H, \Delta T)$ plane shown in Figure 5. We see that, for positive ΔT at 95% CL, the allowed values of Higgs mass extend to well beyond 800 GeV. On

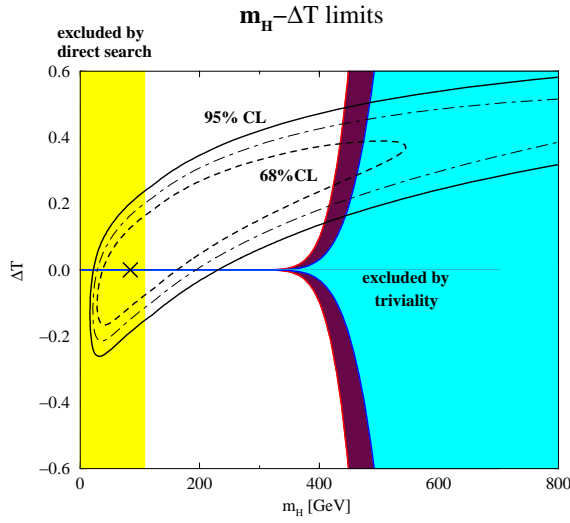


Figure 5: 68% and 95% CL regions allowed¹ in $(m_H, |\Delta T|)$ plane by precision electroweak data.^{2,3} Fit allows for m_t , α_s , and α_{em} to vary consistent with current limits.¹ Also shown by the dot-dash curve is the contour corresponding to $\Delta\chi^2 = 4$, whose intersection with the line $\Delta T = 0$ – at approximately 190 GeV – corresponds to the usual 95% CL upper bound quoted on the Higgs boson mass in the standard model. The triviality bound curves are for $|b|\kappa^2 = 4\pi$ and $4\pi^2$, corresponding to representative models.¹

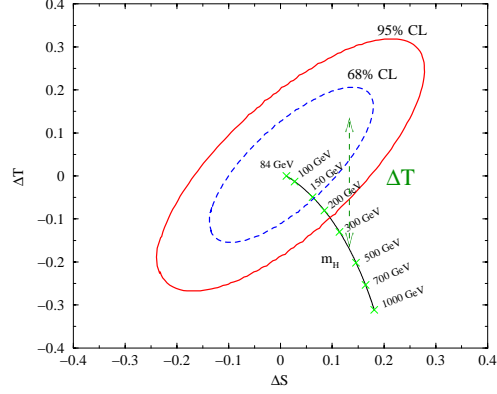


Figure 6: 68% and 95% CL regions allowed in $(\Delta S, \Delta T)$ plane by precision electroweak data.^{2,3} Fit allows for m_t , α_s , and α_{em} to vary consistent with current limits.¹ Standard model prediction for varying Higgs boson mass shown as parametric curve, with m_H varying from 84 to 1000 GeV.

the other hand, not all values can be realized consistent with the bound given in eqn. (4). As shown in figure 5, values of Higgs mass beyond approximately 500 GeV would likely require values of ΔT much larger than allowed by current measurements.

I should emphasize that these estimates are based on dimensional arguments, and we are not arguing that it is *impossible* to construct a composite Higgs model consistent with precision electroweak tests with m_H greater than 500 GeV. Rather, barring accidental cancellations in a theory without a custodial symmetry, contributions to ΔT consistent with eqn. 4 are generally to be expected. Specific composite Higgs boson models are discussed in ref. 1, and the estimates given here are shown to apply.

These results may also be understood by considering limits in the (S, T) plane for *fixed* (m_H, m_t) . In Figure 6, changes from the nominal standard model best fit ($m_H = 84$ GeV) value of the Higgs mass are displayed as contributions to $\Delta S(m_H)$ and $\Delta T(m_H)$. Also shown are the 68% and 95% CL bounds on ΔS and ΔT consistent with current data. We see that, for m_H greater than $\mathcal{O}(200$ GeV), a positive contribution to T can bring the model within the allowed region.

At Run II of the Fermilab Tevatron, it may be possible to reduce the uncertainties in the top-quark and W-boson masses to $\Delta m_t = 2$ GeV and $\Delta M_W = 30$ MeV.¹² Assuming that the measured values of m_t and M_W equal their current central values, such a reduction in uncertainties will result the limits in the $(m_H, \Delta T)$ plane shown in Figure 7. Note that, despite reduced uncertainties, a Higgs mass of up to 500 GeV or so will still be allowed.

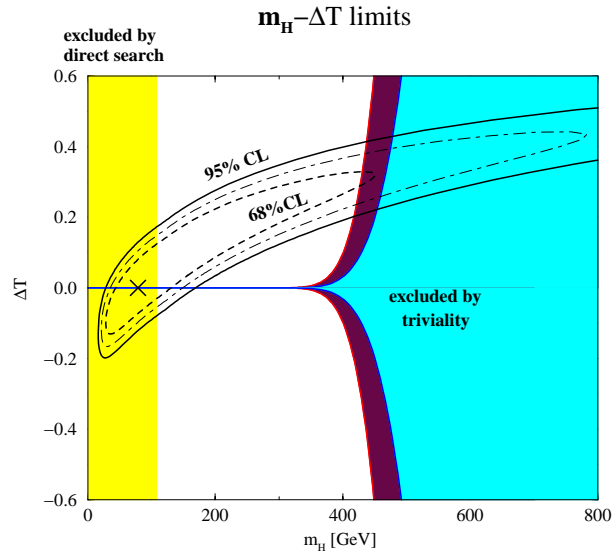


Figure 7: 68% and 95% CL allowed region¹ in $(m_H, \Delta T)$ plane if uncertainty in top-quark and W-boson mass reduce to $\Delta m_t = 2$ GeV and $\Delta M_W = 30$ MeV, as is possible at Run II of the Fermilab Tevatron.

4 Conclusions

In conclusion, the triviality of the Standard Higgs model implies that it is at best a low-energy effective theory valid below a scale Λ characteristic of nontrivial underlying dynamics. As the Higgs mass increases, the upper bound on the scale Λ decreases. If the underlying dynamics does not respect a custodial symmetry, it will give rise to corrections to T of order $\kappa^2 v^2 / \alpha \Lambda^2$, while the contributions to S and U are likely to be much smaller. For this reason, it is necessary to consider limits on a Higgs boson in the $(m_H, \Delta T)$ plane. In doing so, we see that a Higgs mass larger than 200 GeV is consistent with precision electroweak tests if there is a positive ΔT . Absent a custodial symmetry, however, Higgs masses larger than $\simeq 500$ GeV are unlikely: the scale of underlying physics is so low that ΔT is likely to be too large.

5 Acknowledgments

I thank Bogdan Dobrescu, Nick Evans, Christian Hölbling, and E. H. Simmons for fruitful collaborations. This work was supported in part by the U.S. Department of Energy under grant DE-FG02-91ER40676.

References

1. R. S. Chivukula, C. Holbling and N. Evans, “Limits on a composite Higgs boson,” hep-ph/0002022.
2. A. Straessner, talk presented at Recontres de Moriond, March 17, 2000.
3. LEP Electroweak Working Group, <http://lepewwg.web.cern.ch/LEPEWWG/stanmod/lepew99.ps.gz>.
4. LEP Electroweak Working Group, E. Ferrer, talk presented at Recontres de Moriond, March 14, 2000.
5. J. Kuti, L. Lin, and Y. Shen, Phys. Rev. Lett. **61**, 678 (1988).
6. K. G. Wilson, Phys. Rev. **B4**, 3174 and 3184 (1971);
K. G. Wilson and J. Kogut, Phys. Rept. **12**, 75 (1974).
7. M. Weinstein, Phys. Rev. **D8**, 2511 (1973); P. Sikivie, L. Susskind, M. Voloshin, and V. Zakharov, Nucl. Phys. **B173**, 189 (1980).

8. W. Buchmuller and D. Wyler, Nucl. Phys. **B268**, 621 (1986);
B. Grinstein and M. B. Wise, Phys. Lett. **B265**, 326 (1991).
9. A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984);
H. Georgi, Phys. Lett. **B298**, 187 (1993), hep-ph/9207278.
10. M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990) and Phys. Rev. **D46**,
381 (1992).
11. M. Golden and L. Randall, Nucl. Phys. B361 (1991) 3;
B. Holdom and J. Terning, Phys. Lett. B247 (1990) 88;
A. Dobado, D. Espriu and M.J. Herrero, Phys. Lett. B255 (1991) 405.
12. D. Amidei and R. Brock, <http://fnalpubs.fnal.gov/archive/1996/pub/Pub-96-082.ps>.